CSE 110A: Winter 2020

Fundamentals of Compiler Design I

Functions

Owen Arden

UC Santa Cruz

Based on course materials developed by Ranjit Jhala

Functions

Next, we'll build **diamondback** which adds support for

- **User-Defined Functions**

In the process of doing so, we will learn about

- **Static Checking**
- **Calling Conventions**
- **Tail Recursion**

Plan

1. **Defining** Functions
2. **Checking** Functions
3. **Compiling** Functions
4. **Compiling** Tail Calls

1. Defining Functions

First, let’s add functions to our language.

As always, let’s look at some examples.

Example: Increment

For example, a function that increments its input:

```
def incr(x):
  x + 1

incr(10)
```

We have a function definition followed by a single “main” expression, which is evaluated to yield the program’s result, which, in this case, is `11`.

Example: Factorial

Here’s a somewhat more interesting example:

```
def fac(n):
  let t = print(n) in
  if (n < 1):
    1
  else:
    n * fac(n - 1)

fac(5)
```

This program should produce the result

```
5
4
3
2
1
0
120
```

Example: Factorial

Suppose we modify the above to produce intermediate results:

```
def fac(n):
  let t   = print(n)
    , res = if (n < 1):
              1
            else:
              n * fac(n - 1)
  in
    print(res)

fac(5)
```

we should now get:

```
5
4
3
2
1
0
1
1
2
6
24
120
120
```

Example: Mutual Recursion

For this language, **the function definitions are global:** any function can call any other function. This lets us write *mutually recursive* functions like:

```
def even(n):
  if (n == 0):
    true
  else:
    odd(n - 1)

def odd(n):
  if (n == 0):
    false
  else:
    even(n - 1)

let t0 = print(even(0)),
    t1 = print(even(1)),
    t2 = print(even(2)),
    t3 = print(even(3))
in
    0
```

Example: Mutual Recursion

For this language, **the function definitions are global:** any function can call any other function. This lets us write *mutually recursive* functions like:

```
def even(n):
  if (n == 0):
    true
  else:
    odd(n - 1)

def odd(n):
  if (n == 0):
    false
  else:
    even(n - 1)

let t0 = print(even(0)),
    t1 = print(even(1)),
    t2 = print(even(2)),
    t3 = print(even(3))
in
    0
```

What should be the result of executing this program?

Bindings

Lets create a special type that represents places where **variables are bound**,

```
data Bind a = Bind Id a
```

A `Bind` is basically just an `Id` *decorated with* an `a` which will let us save extra *metadata* like **tags** or **source positions** to help report errors

We will use `Bind` at two places:

1. Let-bindings,
2. Function parameters.

It will be helpful to have a function to extract the `Id` corresponding to a `Bind`

```
bindId :: Bind a -> Id
bindId (Bind x _) = x
```

Programs and Declarations

A **program** is a list of declarations and *main* expression.

```
data Program a = Prog
  { pDecls :: [Decl a]    -- ^ function declarations
  , pBody  :: !(Expr a)   -- ^ "main" expression
  }
```

Each **function** lives is its own **declaration**,

```
data Decl a = Decl
  { fName  :: (Bind a)    -- ^ name
  , fArgs  :: [Bind a]    -- ^ parameters
  , fBody  :: (Expr a)    -- ^ body expression
  , fLabel :: a           -- ^ metadata/tag
  }
```

Expressions

Finally, lets add *function application* (calls) to the source expressions:

```
data Expr a
  = ...
  | Let     (Bind a) (Expr a)  (Expr a) a
  | App     Id       [Expr a]           a
```

An *application* or *call* comprises

- an `Id`, the name of the function being called,
- a list of expressions corresponding to the parameters, and
- a metadata/tag value of type `a`.

(**Note:** that we are now using `Bind` instead of plain `Id` at a `Let`.)

Examples Revisited

Finally, lets add *function application* (calls) to the source expressions:

```
data Expr a
  = ...
  | Let     (Bind a) (Expr a)  (Expr a) a
  | App     Id       [Expr a]           a
```

An *application* or *call* comprises

- an `Id`, the name of the function being called,
- a list of expressions corresponding to the parameters, and
- a metadata/tag value of type `a`.

(**Note:** that we are now using `Bind` instead of plain `Id` at a `Let`.)

Examples Revisited

Lets see how the examples above are represented:

```
ghci> parseFile "tests/input/incr.diamond"
Prog {pDecls = [Decl { fName = Bind "incr" ()
                     , fArgs = [Bind "n" ()]
                     , fBody = Prim2 Plus (Id "n" ()) (Number 1 ()) ()
                     , fLabel = ()}
               ]
     , pBody = App "incr" [Number 5 ()] ()
     }
```

```
ghci> parseFile "tests/input/fac.diamond"
Prog { pDecls = [ Decl {fName = Bind "fac" ()
                , fArgs = [Bind "n" ()]
                , fBody = Let (Bind "t" ()) (Prim1 Print (Id "n" ()) ())
                        (If (Prim2 Less (Id "n" ()) (Number 1 ()) ())
                           (Number 1 ())
                           (Prim2 Times (Id "n" ())
                              (App "fac" [Prim2 Minus (Id "n" ()) (Number 1 ()) ()] ())
                              ()) ()) ()
                , fLabel = ()}
                ]
     , pBody  = App "fac" [Number 5 ()] ()
     }
```

2. Static Checking

Next, we will look at an *increasingly important* aspect of compilation, **pointing out bugs in the code at compile time**

Called **Static Checking** because we do this *without* (i.e. *before*) compiling and running ("dynamicking") the code.

There is a huge spectrum of checks possible:

- Code Linting jslint, hlint
- Static Typing
- Static Analysis
- Contract Checking
- Dependent or Refinement Typing

Increasingly, *this* is the most important phase of a compiler, and modern compiler engineering is built around making these checks lightning fast. For more, see this interview of Anders Hejlsberg the architect of the C# and TypeScript compilers.

Static Well-formedness Checking

Suppose you tried to compile:

```
def fac(n):
  let t = print(n) in
  if (n < 1):
    1
  else:
    n * fac(m - 1)

fact(5) + fac(3, 4)
```

We would like compilation to fail, not silently, but with useful messages:

```
$ make tests/output/err-fac.result
```

```
Errors found!

tests/input/err-fac.diamond:6:13-14:
Unbound variable 'm'

         6|      n * fac(m - 1)

tests/input/err-fac.diamond:8:1-9:
Function 'fact' is not defined

         8|  fact(5) + fac(3, 4)
             ^^^^^^^^

tests/input/err-fac.diamond:(8:11)-
(9:1): Wrong arity of arguments at
call of fac

         8|  fact(5) + fac(3, 4)
                       ^^^^^^^^^
```

Static Well-formedness Checking

We get *multiple* errors:

1. The variable `m` is not defined,
2. The function `fact` is not defined,
3. The call `fac` has the wrong number of arguments.

Next, let's see how to update the architecture of our compiler to support these and other kinds of errors.

Types

An *error message* type:

```
data UserError = Error
  { eMsg  :: !Text
  , eSpan :: !SourceSpan
  }
  deriving (Show, Typeable)
```

We make it an *exception* (that can be *thrown*):

```
instance Exception [UserError]
```

Types

We can **create** errors with:

```
mkError :: Text -> SourceSpan -> Error
mkError msg l = Error msg l
```

We can **throw** errors with:

```
abort :: UserError -> a
abort e = throw [e]
```

Types

We **display errors** with:

```
renderErrors :: [UserError] -> IO Text
```

which takes something like:

```
Error
 "Unbound variable 'm'"
 { file      = "tests/input/err-fac"
 , startLine = 8
 , startCol  = 1
 , endLine   = 8
 , endCol    = 9
 }
```

and produce a pretty message (that requires reading the source file),

```
tests/input/err-fac.diamond:6:13-14: Unbound variable 'm'

         6|      n * fac(m - 1)
                         ^
```

Types

We can put it all together by

```
main :: IO ()
main = runCompiler `catch` esHandle

esHandle :: [UserError] -> IO ()
esHandle es = renderErrors es >>= hPutStrLn stderr >> exitFailure
```

Which runs the compiler and if any `UserError` are thrown, `catch`-es and renders the result.

Transforms

Next, lets insert a `checker` phase into our pipeline:

Text $\xrightarrow{\text{Parse}}$ BareP $\xrightarrow{\text{Check}}$ BareP $\xrightarrow{\text{Norm.}}$ AnfP $\xrightarrow{\text{Tag}}$ AnfTagP $\xrightarrow{\text{CodeGen}}$ Asm

In the above, we have defined the types:

```
type BareP   = Program SourceSpan -- ^ sub-expressions have src position metadata
type AnfP    = Program SourceSpan        -- ^ each function body in ANF
type AnfTagP = Program (SourceSpan, Tag) -- ^ each sub-expression has unique tag
```

Catching Multiple Errors

To make using a language and compiler pleasant, we should return *as many errors as possible* in each run.

- Its rather irritating to get errors one-by-one.

We will implement this by writing the functions

```
wellFormed  :: BareProgram -> [UserError]
```

which will *recursively walk over* the entire program, declaration and expression and return the *list of all errors*.

- If this list is empty, we just return the source unchanged,
- Otherwise, we `throw` the list of found errors (and exit.)

Thus, our `check` function looks like this:

```
check :: BareProgram -> BareProgram
check p = case wellFormed p of
            [] -> p
            es -> throw es
```

Well-formed Programs

The bulk of the work is done by:

```
wellFormed :: BareProgram -> [UserError]
wellFormed (Prog ds e)
  =  duplicateFunErrors ds
  ++ concatMap (wellFormedD fEnv) ds
  ++ wellFormedE fEnv emptyEnv e
  where
    fEnv  = fromListEnv [(bindId f, length xs)
                          | Decl f xs _ _ <- ds]
```

This function,

1. **creates** `fEnv`, a map from *function-names* to the *function-arity* (number of params),
2. **computes** the errors for each declaration (given functions in `fEnv`),
3. **concatenates** the resulting lists of errors.

Traversals

Lets look at how we might find three types of errors:

1. “unbound variables”
2. “undefined functions”

(In your assignment, you will look for many more.)

The helper function `wellFormedD` creates an *initial* variable environment `vEnv` containing the functions parameters, and uses that (and `fEnv`) to walk over the body-expressions.

```
wellFormedD :: FunEnv -> BareDecl -> [UserError]
wellFormedD fEnv (Decl _ xs e _) = wellFormedE fEnv vEnv e
  where
    vEnv                         = addsEnv xs emptyEnv
```

Traversals

The helper function `wellFormedE` starts with the input `vEnv0` (which has just) the function parameters, and `fEnv` that has the defined functions, and traverses the expression:

- At each **definition** `Let x e1 e2`, the variable `x` is added to the environment used to check `e2`,
- At each **use** `Id x` we check if `x` is in `vEnv` and if not, create a suitable `UserError`
- At each **call** `App f es` we check if `f` is in `fEnv` and if not, create a suitable `UserError`.

Traversals

```
wellFormedE :: FunEnv -> Env -> Bare -> [UserError]
wellFormedE fEnv vEnv0 e      = go vEnv0 e
  where
    gos vEnv es               = concatMap (go vEnv) es
    go _    (Boolean {})      = []
    go _    (Number  n     l) = []
    go vEnv (Id      x     l) = unboundVarErrors vEnv x l
    go vEnv (Prim1 _ e     _) = go  vEnv e
    go vEnv (Prim2 _ e1 e2 _) = gos vEnv [e1, e2]
    go vEnv (If   e1 e2 e3 _) = gos vEnv [e1, e2, e3]
    go vEnv (Let x e1 e2   _) = go vEnv e1
                             ++ go (addEnv x vEnv) e2
    go vEnv (App f es      l) = unboundFunErrors fEnv f l
                             ++ gos vEnv es
```

You should understand the above and be able to easily add extra error checks.

Quiz

Which function(s) would we have to modify to add *large number errors* (i.e. errors for numeric literals that may cause overflow)?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform-ind

Quiz

Which function(s) would we have to modify to add *large number errors* (i.e. errors for numeric literals that may cause overflow)?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform-grp

Quiz

Which function(s) would we have to modify to add *variable shadowing errors*?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform2-ind

Quiz

Which function(s) would we have to modify to add *variable shadowing errors*?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform2-grp

Quiz

Which function(s) would we have to modify to add *duplicate parameter errors* ?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform3-ind

Quiz

Which function(s) would we have to modify to add *duplicate parameter errors* ?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform3-grp

Quiz

Which function(s) would we have to modify to add *duplicate function errors* ?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform4-ind

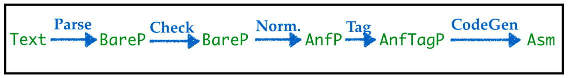
Quiz

Which function(s) would we have to modify to add *duplicate function errors* ?

A. `wellFormed :: BareProgram -> [UserError]`
B. `wellFormedD :: FunEnv -> BareDecl -> [UserError]`
C. `wellFormedE :: FunEnv -> Env -> Bare -> [UserError]`
D. `1` and `2`
E. `2` and `3`

http://tiny.cc/cse110a-wellform4-grp

Compiling Functions



$\text{Text} \xrightarrow{\text{Parse}} \text{BareP} \xrightarrow{\text{Check}} \text{BareP} \xrightarrow{\text{Norm.}} \text{AnfP} \xrightarrow{\text{Tag}} \text{AnfTagP} \xrightarrow{\text{CodeGen}} \text{Asm}$

In the above, we have defined the types:

```haskell
type BareP   = Program SourceSpan -- ^ sub-expressions have src position metadata
type AnfP    = Program SourceSpan        -- ^ each function body in ANF
type AnfTagP = Program (SourceSpan, Tag) -- ^ each sub-expression has unique tag
```

Tagging

$\text{Text} \xrightarrow{\text{Parse}} \text{BareP} \xrightarrow{\text{Check}} \text{BareP} \xrightarrow{\text{Norm.}} \text{AnfP} \xrightarrow{\text{Tag}} \text{AnfTagP} \xrightarrow{\text{CodeGen}} \text{Asm}$

The `tag` phase simply recursively tags each function body and the main expression

ANF Conversion

$\texttt{Text} \xrightarrow{\text{Parse}} \texttt{BareP} \xrightarrow{\text{Check}} \texttt{BareP} \xrightarrow{\text{Norm.}} \texttt{AnfP} \xrightarrow{\text{Tag}} \texttt{AnfTagP} \xrightarrow{\text{CodeGen}} \texttt{Asm}$

- The `normalize` phase (i.e. `anf`) is recursively applied to each function body.
- In addition to `Prim2` operands, each call's arguments should be transformed into an `immediate expression` (Why?)

Generalize the strategy for *binary* operators from Boa

- from (`2` arguments) to `n`-arguments.

Strategy

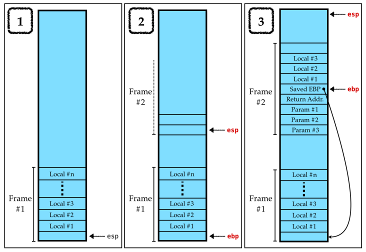
$\texttt{Text} \xrightarrow{\text{Parse}} \texttt{BareP} \xrightarrow{\text{Check}} \texttt{BareP} \xrightarrow{\text{Norm.}} \texttt{AnfP} \xrightarrow{\text{Tag}} \texttt{AnfTagP} \xrightarrow{\text{CodeGen}} \texttt{Asm}$

Now, let's look at *compiling* function *definitions* and *calls*. We need a co-ordinated strategy.

Definitions — Each *definition* is compiled into a labeled block of `Asm` that implements the *body* of the definitions. (But what about the *parameters*)?

Calls — Each *call* of `f(args)` will execute the block labeled `f` (But what about the *parameters*)?

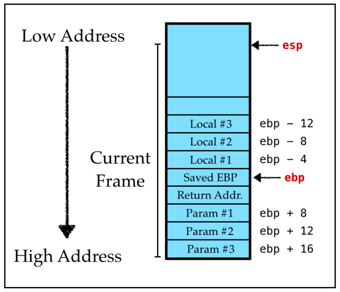
Strategy: The Stack



We will use our old friend, *the stack* to

- pass *parameters*
- have *local variables* for called functions

Calling Convention



Recall that we are using the `C` calling convention that ensures the above stack layout

---

Strategy: Definitions

When the function body starts executing, the parameters `x1`, `x2`, … `xn` are at `[ebp + 4*2]`, `[ebp + 4*3]`, … `[ebp + 4*(n+1)]`.

1. Ensure that enough stack space is *allocated* i.e. that `esp` and `ebp` are properly managed
2. Compile body with *initial* `Env` mapping parameters to `-2`, `-3`, …, `-(n+1)`.

Strategy: Calls

As in Cobra, we must ensure that the parameters actually live at the above address.

1. *Before* the call, `push` the parameter values onto the stack in reverse order,
2. *Call* the appropriate function (using its label),
3. *After* the call, *clear* the stack by incrementing `esp` appropriately.

NOTE:
At both *definition* and *call*, if you are compiling on MacOS, you need to also respect the 16-Byte Stack Alignment Invariant

Types

We already have most of the machinery needed to compile calls.

Lets just add a new kind of `Label` for each user-defined function:

```
data Label
  = ...
  | DefFun Id
```

We will also extend the `Arg` type to include information about size directives

```
data Arg
  = ...
  | Sized Size Arg
```

We will often need to specify that an `Arg` is a *double word*
(the other possibilities are - single `word` and `byte`) which we needn't worry about.

```
data Sized
  = DWordPtr
```

Implementation

Lets can refactor our `compile` functions into:

```
compileProg ::        AnfTagP -> Asm
compileDecl ::        AnfTagD -> Asm
compileExpr :: Env -> AnfTagE -> Asm
```

that respectively compile `Program`, `Decl` and `Expr`.

In order to **simplify stack management as in Cobra** lets have a helper function that compiles the *body* of each function:

```
compileBody :: Env -> AnfTagE -> Asm
```

`compileBody env e` will wrap the `Asm` generated by `compileExpr env e` with the code that manages `esp` and `ebp`.

Compiling Programs

To compile a `Program` we compile each `Decl` and the main body expression

```
compileProg (Prog ds e)
  =  compileBody emptyEnv e
  ++ concatMap   compileDecl ds
```

QUIZ: Does it matter whether we put the code for `e` before `ds`?

1. Yes
2. No

Compiling Programs

To compile a `Program` we compile each `Decl` and the main body expression

```
compileProg (Prog ds e)
  =  compileBody emptyEnv e
  ++ concatMap   compileDecl ds
```

QUIZ: Does it matter what order we compile the `ds` ?

1. Yes
2. No

Compiling Declarations

To compile a single `Decl` we

1. Create a block starting with a label for the function's name (so we know where to `call`),
2. Invoke `compileBody` to fill in the assembly code for the body, using the initial `Env` obtained from the function's formal parameters.

```
compileDecl :: ADcl -> [Instruction]
compileDecl (Decl f xs e _)
  = ILabel (DefFun (bindId f))
  : compileBody (paramsEnv xs) e
```

Compiling Declarations

The initial `Env` is created by `paramsEnv` which returns an `Env` mapping each parameter to its stack position

```
paramsEnv :: [Bind a] -> Env
paramsEnv xs = fromListEnv (zip xids [-2, -3..])
  where
    xids     = map bindId xs
```

(Recall that `bindId` extracts the `Id` from each `Bind`)

Compiling Declarations

Finally, as in cobra, `compileBody env e` wraps the assmbly for `e` with the code that manages `esp` and `ebp`.

```
compileBody :: Env -> AnfTagE -> Asm
compileBody env e = entryCode e
                 ++ compileExpr env e
                 ++ exitCode e
                 ++ [IRet]

entryCode :: AnfTagE -> Asm
entryCode e = [ IPush (Reg EBP)
              , IMov  (Reg EBP) (Reg ESP)
              , ISub  (Reg ESP) (Const 4 * n)
              ]
  where
    n       = countVars e

exitCode :: AnfTagE -> Asm
exitCode = [ IMov (Reg ESP) (Reg EBP)
           , IPop (Reg EBP)
           ]
```

Compiling Calls

Finally, lets extend code generation to account for calls:

```
compileExpr :: Env -> AnfTagE -> [Instruction]
compileExpr env (App f vs _)
  = call (DefFun f) [param env v | v <- vs]
```

The function `param` converts an **immediate expressions** (corresponding to function arguments)

```
param :: Env -> ImmE -> Arg
param env v = Sized DWordPtr (immArg env v)
```

The `Sized DWordPtr` specifies that each argument will occupy a double word (i.e. 4 bytes) on the stack.

EXERCISE

The hard work compiling calls is done by:

```
call :: Label -> [Arg] ->
[Instruction]
```

Fill in the implementation of `call` yourself. As an example of its behavior, consider the (source) program:

```
def add2(x, y):
  x + y

add2(12, 7)
```

The call `add2(12, 7)` is represented as:

```
App "add2" [Number 12, Number 7]
```

The code for the call is generated by

```
call (DefFun "add2") [arg 12, arg 7]
```

where `arg` converts source values into assembly `Arg` which *should* generate the equivalent of the assembly:

```
push DWORD 14
push DWORD 24
call label_def_add2
add esp, 8
```

Compiling Tail Calls

Our language doesn't have *loops*. While recursion is more general, it is more *expensive* because it uses up stack space (and requires all the attendant management overhead). For example (the `python` program):

```
def sumTo(n):
  r = 0
  i = n
  while (0 <= i):
    r = r + i
    i = i - 1
  return r

sumTo(10000)
```

- Requires a *single* stack frame
- Can be implemented with 2 registers

But, the "equivalent" `diamond` program

```
def sumTo(n):
  if (n <= 0):
    0
  else:
    n + sumTo(n - 1)

sumTo(10000)
```

- Requires `10000` stack frames …
- One for `fac(10000)`, one for `fac(9999)` etc.

Tail Recursion

Fortunately, we can do much better.

A **tail recursive** function is one where the recursive call is the *last* operation done by the function, i.e. where the value returned by the function is the *same* as the value returned by the recursive call.

We can rewrite `sumTo` using a tail-recursive `loop` function:

```
def loop(r, i):
  if (0 <= i):
    let rr = r + i
      , ii = i - 1
    in
      loop(rr, ii)   # tail call
  else:
    r

def sumTo(n):
  loop(0, n)

sumTo(10000)
```

Visualizing Tail Calls

```
sumTo(5)
==> 5 + sumTo(4)
        ^^^^^^^^

==> 5 + [4 + sumTo(3)]
             ^^^^^^^^

==> 5 + [4 + [3 + sumTo(2)]]
                  ^^^^^^^^

==> 5 + [4 + [3 + [2 + sumTo(1)]]]
                       ^^^^^^^^

==> 5 + [4 + [3 + [2 + [1 + sumTo(0)]]]]
                            ^^^^^^^^

==> 5 + [4 + [3 + [2 + [1 + 0]]]]
                        ^^^^^

==> 5 + [4 + [3 + [2 + 1]]]
                   ^^^^^

==> 5 + [4 + [3 + 3]]
              ^^^^^

==> 5 + [4 + 6]
         ^^^^^

==> 5 + 10
    ^^^^^^
==> 15
```

Plain Recursion

- Each call **pushes a frame** onto the call-stack;
- The results are **popped off** and *added* to the parameter at that frame.

Visualizing Tail Calls

```
sumTo(5)
==> loop(0, 5)
==> loop(5, 4)
==> loop(9, 3)
==> loop(12, 2)
==> loop(14, 1)
==> loop(15, 0)
==> 15
```

Tail Recursion

- Accumulation happens in the parameter (not with the output),
- Each call returns its result *without further computation*

No need to use call-stack, can make recursive call **in place**. * Tail recursive calls can be *compiled into loops*!

Tail Recursion Strategy

Instead of using `call` to make the call, simply:

1. **Move** the *call’s* arguments to the (same) stack position (as current args),
2. **Free** current stack space by resetting `esp` and `ebp` (as just prior to `ret` c.f. `exitCode`),
3. **Jump** to the *start* of the function.

That is, here’s what a *naive* implementation would look like:

```
push [ebp - 8]         # push ii
push [ebp - 4]         # push rr
call def_loop
```

Tail Recursion Strategy

but a *tail-recursive* call can instead be compiled as:

```
mov eax , [ebp - 8]  # overwrite i with ii
mov [ebp + 12], eax
mov eax, [ebp - 4]   # overwrite r with rr
mov [ebp + 8], eax
mov esp, ebp         # "free" stack frame (as before `ret`)
pop ebp
jmp def_loop         # jump to function start
```

which has the effect of executing `loop` *literally* as if it were a while-loop!

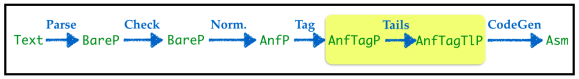
Requirements

To *implement* the above strategy, we need a way to:

1. **Identify** tail calls in the source `Expr` (AST),
2. **Compile** the tail calls following the above strategy.

Types

We can do the above in a single step, i.e., we could identify the tail calls during the code generation, but its cleaner to separate the steps into:



$$\texttt{Text} \xrightarrow{\text{Parse}} \texttt{BareP} \xrightarrow{\text{Check}} \texttt{BareP} \xrightarrow{\text{Norm.}} \texttt{AnfP} \xrightarrow{\text{Tag}} \texttt{AnfTagP} \xrightarrow{\text{Tails}} \texttt{AnfTagTlP} \xrightarrow{\text{CodeGen}} \texttt{Asm}$$

In the above, we have defined the types:

```
type BareP    = Program SourceSpan -- ^ sub-expressions have src position metadata
type AnfP     = Program SourceSpan        -- ^ each function body in ANF
type AnfTagP = Program (SourceSpan, Tag) -- ^ each sub-expression has unique tag
type AnfTagTlP = Program ((SourceSpan, Tag), Bool)
    - ^ each call is marked as "tail" or not
```

Transforms

Thus, to implement tail-call optimization, we need to write *two* transforms:

1. **To Label** each call with `True` (if it is a *tail call*) or `False` otherwise:

```
tails :: Program a -> Program (a, Bool)
```

2. **To Compile** tail calls, by extending `compileExpr`

Labeling Tail Calls

```
def facTR(acc, n):
  if (n < 1):
    acc
  else:
    if (n == 2):
      2 ∗ facTR(n – 1, n – 1) Not Tail
    else:
     facTR(acc ∗ n, n – 1) Tail
```

```
data Expr
  = Number  Integer
  | Boolean Bool
  | Id      Id
  | Prim1   Prim1  Expr
  | Prim2   Prim2  Expr  Expr
  | If      Expr   Expr  Expr
  | Let     Bind   Expr  Expr
  | App     Id    [Expr]
```

The `Expr` in *non tail positions*

- `Prim1`
- `Prim2`
- `Let` ("bound expression")
- `If` ("condition")

cannot contain tail calls; all those values have some further computation performed on them.

Labeling Tail Calls

```
def facTR(acc, n):
  if (n < 1):
    acc
  else:
    if (n == 2):
      2 ∗ facTR(n – 1, n – 1) Not Tail
    else:
     facTR(acc ∗ n, n – 1) Tail
```

```
data Expr
  = Number  Integer
  | Boolean Bool
  | Id      Id
  | Prim1   Prim1  Expr
  | Prim2   Prim2  Expr  Expr
  | If      Expr   Expr  Expr
  | Let     Bind   Expr  Expr
  | App     Id    [Expr]
```

However, the `Expr` in *tail positions*

- `If` ("then" and "else" branch)
- `Let` ("body")

can contain tail calls (*unless* they appear under the first case)

Transforms

Algorithm: Traverse `Expr` using a `Bool`

- Initially `True` but
- Toggled to `False` under *non-tail positions*,
- Used as "tail-label" at each call.

NOTE: All non-calls get a default tail-label of `False`.

Transforms

```
tails :: Expr a -> Expr (a, Bool)
tails = go True                                  -- initially flag is True
  where
    noTail l z              = z (l, False)
    go _ (Number n l)       = noTail l (Number n)
    go _ (Boolean b l)      = noTail l (Boolean b)
    go _ (Id     x l)       = noTail l (Id x)
    go _ (Prim2 o e1 e2 l)  = noTail l (Prim2 o e1' e2')
      where
        [e1', e2']          = go False <$> [e1, e2]   -- "prim-args" is non-tail
    go b (If c e1 e2 l)     = noTail l (If c' e1' e2')
      where
        c'                  = go False c             -- "cond" is non-tail
        e1'                 = go b     e1            -- "then" may be tail
        e2'                 = go b     e2            -- "else" may be tail
    go b (Let x e1 e2 l)    = noTail l (Let x e1' e2')
      where
        e1'                 = go False e1            -- "bound-expr" is non-tail
        e2'                 = go b     e2            -- "body-expr" may be tail
    go b (App f es l)       = App f es' (l, b)       -- tail-label is current flag
      where
        es'                 = go False <$> es        -- "call args" are non-tail
```

Transforms

```
tails :: Expr a -> Expr (a, Bool)
tails = go True                                  -- initially flag is True
  where
    noTail l z              = z (l, False)
    go _ (Number n l)       = noTail l (Number n)
    go _ (Boolean b l)      = noTail l (Boolean b)
    go _ (Id     x l)       = noTail l (Id x)
    go _ (Prim2 o e1 e2 l)  = noTail l (Prim2 o e1' e2')
      where
        [e1', e                                          " is non-tail
    go b (If c
      where
        c'                                           on-tail
        e1'                                          e tail
        e2'                 = go b     e2            -- "else" may be tail
    go b (Let x e1 e2 l)    = noTail l (Let x e1' e2')
      where
        e1'                 = go False e1            -- "bound-expr" is non-tail
        e2'                 = go b     e2            -- "body-expr" may be tail
    go b (App f es l)       = App f es' (l, b)       -- tail-label is current flag
      where
        es'                 = go False <$> es        -- "call args" are non-tail
```

EXERCISE: How could we modify tails to *only* mark **tail-recursive** calls, i.e. to the *same* function (whose declaration is being compiled?)

Compiling Tail Calls

Finally, to generate code, we need only add a special case to `compileExpr`

```
compileExpr :: Env -> AnfTagTlE -> [Instruction]
compileExpr env (App f vs l)
  | isTail l        = tailcall (DefFun f) [param env v | v <- vs]
  | otherwise       = call     (DefFun f) [param env v | v <- vs]
```

That is, *if* the call is *not labeled* as a tail call, generate code as before. Otherwise, use `tailcall` which implements our tail recursion strategy

```
tailcall :: Label -> [Arg] -> [Instruction]
tailcall f args
  = moveArgs args   -- overwrite current param slots with call args
 ++ exitCode        -- restore ebp and esp
 ++ [IJmp f]        -- jump to start
```

EXERCISE: Does the above strategy work *always*? Can you think of situations where it may go horribly wrong?